ANALYSIS OF EARLY-AGE CRACKING FOR UHPFRC ELEMENTS (CAST-IN-PLACE OR PREFABRICATED) BASED ON EXPERIENCE OF GLENMORE / “PASSERELLE DES ANGES” FOOTBRIDGES

L. Sorelli (1), F. Toutlemonde (2), Ulm, F.-J. (3), Vic Perry (4) and Dominique Corvez (4), A. Sheikh (1)

(1) Université Laval, 1065 av. de la Médecine, PLT2928a, G1V 0A6, Quebec, QC, Canada
(2) IFSTTAR, Paris - 58 boulevard Lefebvre 75732 Paris Cedex 15 France
(3) Massachusetts Institute of Technology, Cambridge, USA
(3) Lafarge Ductal®, Paris, France

Abstract
This work presents a model-based analysis of the risk of early age cracking in structures made of Ultra High Performance Concrete (UHPC). First, we summarize the background works on modelling UHPC at early age with emphasis on the thermodynamics hypothesis of partial decoupling which substantially simplifies the application and calibration of the model. Then, after calibrating the chemo-thermo-mechanical properties, we present 2 case studies: the UHPC footbridge in Calgary, Canada, and the footbridge of the Angle in Montpellier, France. The comparison between the simulated and measured histories of temperature and strains is presented. The concept of level of loading was employed to assess the risk of early age cracking. Finally, a probabilistic analysis of a simplified 1D case is employed to support the final conclusions.

INTRODUCTION
Ultra High Performance Concretes (UHPC) are emerging in civil engineering thanks to their outstanding properties, such as very high compressive strength, tensile ductility, and an extremely low permeability [1-2]. However, UHPC presents new engineering challenges such as the risk of early age cracking. For instance, prototype bridge elements cast at a precast plant in Lexington, Kentucky, were used to develop and model casting, curing and demoulding methods. Fig 1a shows elements with severe cracking at the first days after casting. The low volume-to-cement ratio of UHPC involves significant thermal deformations and shrinkage. The origin of the latter is twofold: the chemical shrinkage, which is due to the volumetric imbalance between the products and the reactants of the hydration (known as the Le Chatelier’s effect), followed by the autogeneous shrinkage, which is an additional contraction due to an increase of capillary tension in de-saturating pores. If the tensile stress that is induced by restrained deformations overcame the aging tensile strength, cracks occur [3]. Experimental observations showed that UHPC autogenous shrinkage starts after a dormant period of 10-30 hours, and reaches an asymptotic value of 400-800 µm/m at about 10 days, depending on the mix composition and the external conditions [4].
To estimate of the effect of latent heat of hydration, one can evaluate the characteristic length \( \ell_{ch} \) of the heat hydration-diffusion equation is evaluated [5]. Fig.1 shows that the \( \ell_{ch} \) for UHPC is about 20-30 cm, which is smaller than that of Normal and High Strength Concrete (NC and HSC). For small structure size \( L << \ell_{ch} \) the hydration latent heat is evacuated without significant temperature rise, larger UHPC structures may ask for a proper analysis of the local thermo-activation effect on structural stability. From a design standpoint, engineers could increase the outwards heat flow by increasing the surface-to-volume ratio (S/V) or reducing the thermal conductivity-to-exchange coefficient ratio (K/\( \lambda_{ch} \)) with a proper choice of material formwork.

![Figure 1. Cracks in prototype bridge elements used to model casting, curing and demoulding for a bridge project.](image)

This work presents model-based simulations of the construction phases of two UHPC footbridges constructed out by Lafarge in Calgary, Canada, and in Montpellier, France. In the following, we briefly present the model features [5-8] with emphasis on the hypothesis of partial decoupling which facilitates the calibration of the model parameters without losing in accuracy of the on-site measurements’ prediction of the. The presented results emphasize the key role of the chemical affinity as material-to-structure parameter and the levels of loading as risk indicator of early age cracking.

2. UHPC MODELING AT EARLY AGE

2.1 One-dimensional thought model for hardened state

Chuang and Ulm proposed a two-phase model to reproduce the quasi-ductile UHPC response under tension [9]. The rheological model sketched in Fig.2.a consists of two parallel sub-devices coupled by an elastic spring. The brittle behaviour of the matrix is reproduced by the spring of stiffness \( C_M \) in series with a brittle crack device of strength \( f_t \) and the frictional element of strength \( k_M \). The ductile behaviour of the fibres is described by the spring with stiffness \( C_F \) and the frictional element of strength \( f_y \). To capture the fibre-matrix bond behaviour, the matrix plastic strain \( \varepsilon_{pM} \) is coupled with the fibre plastic strain \( \varepsilon_{pF} \) by means of an additional spring M. The matrix stress \( \sigma_{M} \) and the fibre stress \( \sigma_{F} \) act, respectively, on the spring \( C_M \) and on the spring \( C_F \). The overall stress-strain relationship, \( \sigma - \varepsilon \), under tensile loading is shown in Fig.2.b: until the stress reaches the tensile strength \( \sigma_t \), the response is
elastic with stiffness $K_0$; then, first cracking causes a stress drop to $\sigma_1$, the post-cracking behaviour is still linear with stiffness $K_1$ until a strength $\sigma_2$. Note the model disregards tension softening behaviour occurring at relatively large strains of about 0.2-0.3%. The model has been implemented in the code CESAR as the EAHC module [10].

Figure 2 (Left) Rheological model for aging UHPC materials [7]; (Right) Calibration of the material parameters on experimental tensile tests.

### 2.2 Three-dimensional thermo-chemo-mechanical model

The reaction degree $r = m(t)/m(\infty) \in [0-1]$ is defined as the ratio between the hydrated water $m(t)$ and the asymptotic hydrated water $m(\infty)$ [6]. The starting point of the thermo-chemical model is replacing a second order expansion of the free energy $\psi = \psi(e, r, T)$ within the classical Clasius-Duhem inequality $\sigma dt = \sigma dT - \varphi dS - SdT \geq 0$; where $\sigma dt$ is the dissipated energy, $\sigma = \sigma_\varphi + \sigma_\psi$, $S$ is the entropy and $T$ is the temperature. Assuming the partial decoupling that reaction affinity and entropy are not affected by external loads, the resulting incremental state equations can be decoupled in the following 2 sub-systems:

$$ d\sigma = d\sigma_\varphi + d\sigma_p = C(r) : \left( d\varepsilon_p - d\varepsilon_p^r \right) = C(r) : \alpha dT - C_M(r) : \beta dr $$

(1)

$$ \left\{ \frac{dS}{dA} = \begin{bmatrix} -\frac{C}{T_0} & -\frac{1}{T_0} \\ 0 & -\eta \end{bmatrix} \right\} \left\{ \begin{bmatrix} dT \\ dr \end{bmatrix} \right\} $$

(2)

where $C(r)$ is the elastic stiffness tensor, $\alpha$ is the thermal expansion coefficient, $\beta$ is a chemical shrinkage coefficient, and $d\varepsilon_p = d\varepsilon_p^e + d\varepsilon_p^r$. For an elastic analysis, one can assume $d\varepsilon_p^e = 0$. The cracking conditions are defined according to the standard framework of multi-surface plasticity (for more details see [8]).

### 2.2 Hydration kinetics

The kinetics law of the 2nd equation of Eq.(2) is recast in the following Arrhenius's form:
where $\tilde{A}(r)$ is the affinity function, and the ratio between energy activation and the constant of perfect gas $E_a/R$ accounts for thermal activation behaviour of concrete hydration.

2.3 Heat Equation

Combining the first and second laws of thermodynamics $TdS = W^{\text{ext}} - d\psi - SdT + \delta Q$ with the first equation of (2) yields the following working equation:

$$C \frac{dT}{dt} = K\nabla^2 T + L \frac{dr}{dt}$$

(4)

where $\psi$ is the free energy, and $\delta Q = \text{div} \mathbf{q}$ is the heat absorbed by the system through the boundary for a linear isotropic conduction law, $\mathbf{q} = -K \nabla T$, where $\nabla T$ is the temperature gradient and $K$ is the (scalar) conductivity coefficient. In energy balance Eq (4), $C dT/dt$ is the energy change stored in the system and $K\nabla^2 T$ is the net heat rate provided from the outside and $L dr/dt$ is the heat generated by hydration. The boundary condition on the heat flux $\mathbf{q}$ through the surface $\partial S$, which is oriented by an outward unit vector $\mathbf{n}$, is:

$$\mathbf{q} \cdot \mathbf{n} = -\lambda (T^{\text{surf}} - T^{\text{ext}})$$

(5)

3. MATERIAL CHARACTERIZATION

2.1 Hardened Property and Strength's and Stiffness' Evolution

The elastic properties $(C_M, M, C_F)$ and strength properties $(f_t, k_M, f_y)$ were calibrated by fitting the experimental tensile response of specimens with dimension $70 \times 70 \times 280$ mm as shown on the right side of Figure 1 [9]. The material parameters for Ductal® are as follows: $C_M = 50'000$ MPa, $C_F = 1.0$ MPa, $M = 1650$ MPa; $f_t = 5.0$ MPa, $k_M = 3.0$ MPa, $f_y = 4.6$ MPa. The strength properties of the double Drucker-Prager loading functions are taken in accordance to available data in literature [6-8]. In a first approach, the material is considered isotropic and the same Poisson's ratio is assumed for steel fibre and cement matrix (0.2).

2.2 Strength Growth and Stiffness Evolution

The Young's modulus evolves with the hardening up to an asymptotic value. For the stiffness evolution, the Byfors' law of Eq (7) is assumed as showed in Figure 3. As for the strength, it was observed that the UHPC tensile strength can develop faster due to the lack of aggregates and it depends strongly on the mix design choice [4]. In absence of experimental data for UHPC Ductal®, we simply assume a linear evolution law for the compressive and tensile strength which is linear, that is $n=1$, as shown in Figure 3. Note that the strength increases from 0 to $f(\infty)$ only after an initial percolation threshold $r_0$. 

$$\frac{dr}{dt} = \exp\left( - \frac{E_a}{RT} \right) \tilde{A}(r)$$

(3)
Stiffness evolution:

\[
\frac{E(r)}{E(\infty)} = \frac{1+1.37 f_c(\infty)^{2.204}}{1+1.37 f_c(r)^{2.204}} \left( \frac{f(\infty)}{f(r)} \right)^{2.675}
\]  
(6)

\[
f(r) = \begin{cases} 
  r_0 f(\infty)/10 & \text{if } r \leq r_0 \\
  (f(r) - f(\infty)) \frac{r - r_0}{1-r_0} + f(\infty) & \text{if } r > r_0 
\end{cases}
\]

Strength evolution:

\[
\frac{f(r)}{f(r=1)} = \left( \frac{r - r_0}{1-r_0} \right)^n
\]  
(7)

Figure 3. Elastic stiffness and strength vs. reaction degree.

2.3 Thermo-chemical properties

Due to the high cement content, the volume heat capacity \( C \) is assumed as \( C=2700 \text{ kJ/m}^3/\text{K} \) [2]. The thermal conductivity, which mainly depends on the density, is about 10 \( \mu \text{m/m/K} \) after the scientific report [2] and the AFGC recommendations [1]. This value is considered constant during all the hydration [11]. The chemical dilatation coefficient \( \beta=450 \mu\text{m/m} \) is equal to the asymptotic strain measured in shrinkage tests on Ductal® specimens in open air [2]. The percolation threshold \( r_0=0.1 \) is a common value in the early-age concrete literature [5]. By considering that the isothermal strength growth \( f_{c,iso} \) is approximately proportional to the increase of hydration degree [5], i.e., \( df_{c,iso} \propto d\xi \), Eq.(3) is employed as a means to access the normalized affinity from the isothermal strength evolution tests. Figure 4. shows the isothermal compressive strength curve interpolating the experimental tests on Ductal® and the corresponding normalized affinity \( \hat{A}(\xi) \), respectively.

Figure 4. (Left) Compressive strength vs. time; (Right). Employed affinity vs. hydration degree.
4. STUDY CASE

4.1 Case Study No.1: The Glenmore Footbridge

The pedestrian bridge in Calgary, Canada, was constructed by Lafarge Canada Inc. on November, 2006 and was the longest precast UHPC piece cast at one time. Due to the size of the monolithic pour (37 m$^3$), the batching and casting procedure lasted 8.8 hours. Figure 4.a shows the lateral view of the footbridge. The footbridge has a length of about 33.6 m and a width of about 3.6 m. The height of the tee-girder section varies from 1.1 m to 1.4 m so that the curved intrados is about 0.3 m higher than the ends. Bars of Glass Fiber Reinforced Polymer (GFRP) were added as redundant shear reinforcement. The prestressing cables are made of multiple strands of 15 mm diameter. 8 thermocouples (TC) and the 4 strain gauges (SG) were placed to measure temperatures and longitudinal strains.

![Figure 5. (a) Lateral view of the Glenmore footbridge; (b) Formwork.](image)

The formwork is shown in Figure 5. The construction phases are summarized in Table 1 during the first 200 hours of curing. The casting procedure lasted 8.8 hours. For the curing, the footbridge was placed in a tent equipped with a heat system.

The decoupling hypothesis allows solving the early-age model in a two step manner. First, the thermo-chemical system of Eqs. (3) and (4) are solved to predict the temperature and reaction degree. Then, the solution of the mechanical problem of Eq. (1) provides the stress and strain field. In the FE model, we consider the prestressing effect as equivalent loading, while we disregard the effect of the GFRP bars on the elastic behaviour of the structure. The average temperature of the material at beginning of casting was about 6°C. Within the module TEXO of the finite element code CESAR-LCPC [10], the hydrating bulk material is modelled by means of solid elements, while the loss of heat through the formwork is modelled by linear exchange elements according to Eq.(5). Formwork removal is modelled by modifying the exchange coefficient $\lambda$ with values from existing literature, such as $\lambda = 9.18$ KJ/hr/m$^2$/K for plywood of 40 mm thickness, $\lambda = 1.55$ KJ/hr/m$^2$/K for polystyrene, $\lambda = 14.4$ KJ/hr/m$^2$/K for steel layer, $\lambda = 14.4$ KJ/hr/m$^2$/K at open air. As for the displacement boundary conditions, no displacement normal to the surface is allowed with formwork in place, while the surface is stress-free once the formwork is taken off. As the supports are inclined upwards by 3.4%, the displacement in the longitudinal direction results restrained. Additionally, flared ends on each end of the girder created shrinkage restraint, which was accommodated by embedding polystyrene, then dissolving following initial set.
Table 1  Construction phases.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Duration</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-74.5</td>
<td>Curing before dissolving polystyrene formwork</td>
</tr>
<tr>
<td>2</td>
<td>74.5-77.8</td>
<td>Curing before prestressing the tendon T#1</td>
</tr>
<tr>
<td>3</td>
<td>77.8-107.0</td>
<td>Curing before removing the rest of the formwork</td>
</tr>
<tr>
<td>4</td>
<td>107.0-131.7</td>
<td>Curing before prestressing tendons T#2 and T#3</td>
</tr>
<tr>
<td>5</td>
<td>131.7-201.6</td>
<td>Curing</td>
</tr>
</tbody>
</table>

Figure 6 shows a comparison between the simulated and measured temperatures over time for the points monitored in mid-span section. The effect of hydration heat is not negligible as the temperature rise is close to the maximum expected for adiabatic condition (C/L=XX). In spite of the model simplifications, the comparison of the simulated results with the measured histories of temperatures and strains is fairly satisfactory.

Figure 7.a compares the measured and simulated strain history for the monitored point of the mid-span section. Strain measurement error is about 24%±2% between 70 and 200 hours. The concept of level of loading was employed to evaluate the risk of early age cracking which measures the distance of a stress state to the loading function from the centre direction [12]. Figure 7 shows the levels of loading for the point in the mid-span section which experienced the highest strain gradient. The level of loading for tension-tension stress of states has reached a maximum of 39% at about 63 hours after casting.

Figure 7. (a) Simulated and experimental strain histories for mid-span section; (b) Evolution over time of the levels of loadings for the critical points in the mid-span section.
4.2 Case Study No.2: The footbridge “Passarelle des Anges”

The model EAHC was applied to study the non linear behavior of the segments of a footbridge under construction in Montpellier, namely “Passarelle des Anges” [13]. One bridge segment cracked at early age during the construction: Fig.8a shows the cracks passing throughout the thickness which are close to the central part and the transversal connection. The model consider 5 casting phases (Fig. 8): casting of bottom part (phase1), central part (phase 2), top part (phase 3), opening of the mold (phase 4), removal of the molding (phase 5). The FE model is composed by 8980 nodes, 13023 solid element, 4246 surface elements for heat exchange, 320 special elements for assuring consistency between the casting phases. The calibration of the model parameters is here omitted as quite similar to the previous section. The thermal boundary condition was simulated with a thermal coefficient of 14.2 KJ/hr/m2/K for steel formwork [5]. The mechanical boundary condition was considered as fully restrained without any normal displacement with respect to the formwork, which is all in place around section up to the first 3 construction phases.

The simulation well predicted the measured temperature history within the sample N.6, although the cooling rate was underestimated (Fig. 9 left side). In this case, to assess the risk of cracking, we compared the stress fields by the linear-elastic and non-linear mechanical models (Fig. 9 right side). The cracking onset occurred at 32 hours and the plastic zone developed in the central zone, as experimentally observed.

Figure 8. Crack observed on the sample N.6 ; Finite Element Mesh employed to analyze the 3 casting phase with the code Cesar TEXO and Cesar EAHC-MEXO.

Other simulations were run to check the effect of an initial pre-heat and the mechanical boundary condition by considering 3 different cases (fully restrained, partially restrained, and minimally restrained). It was found that the risk of cracking strongly depends on the choice of the mechanical boundary conditions to a point that no cracking occur if the sample is minimal restrained. The location of the plastic zone is also affected by the choice of boundary conditions.
5. TOWARDS A PROBABILISTIC MODEL

A probabilistic model was set up based on a simplified cylindrical axial-symmetric formulation of the thermo-chemo-mechanical equations (1) and (2) for a 1D strip bar of 7m length and 0.30m diameter. The cylinder is longitudinally restrained by an elastic spring of stiffness K, which reproduces the structural restraint degree. The input random variables of the model are the material parameters and the external temperature (T\text{env}). The parameters’ mean values are the same of the previous sections, while a coefficient of variation of 10% for all variables is considered. The reliability index of the risk of cracking is defined by Eq. (8), where $m$ is the average value and $\delta$ is the standard deviation of tensile stress ($\sigma$) and tensile strength ($f_t$). Figure 10 shows the results of a Monster Carlo analysis in terms of the reliability index vs. time. The variance decomposition of the variable $m_\sigma(t) - m_{f_t}(t)$ is also showed for 1 day, 3 day and 7 day. These preliminary results show that the dispersion on the energy activation has major effect on the reliability index up to 2 days; afterwards the key parameters are the aging elastic modulus, the restraint stiffness, and the environmental temperature.

$$\beta = \frac{|m_\sigma(t) - m_{f_t}(t)|}{\sqrt{\delta_\sigma^2 + \delta_{f_t}^2 - 2\delta_{f_t}\delta_\sigma}} > 0$$ (8)
5. CONCLUDING REMARKS AND FUTURE OUTLOOK

1. The effect of hydration heat can be critical for UHPC structures with size larger of 20-30 cm, which is the critical diffusion length;
2. The partial de-coupling hypothesis facilitates the calibration of the model properties with a satisfactory prediction of temperature and strain histories;
3. The concept of level of loading was used as indicator of risk of early age cracking. Furthermore, a preliminary probabilistic approach was presented as powerful tool to identify the impact of material parameters on the risk of cracking;
4. Future works will attempt to include concrete creep which has been disregarded in this work resulting in overestimation of the risk of cracking.

REFERENCES