h e p i a
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Devoir n°5 – Calcul des structures
GC2 – Année académique 2011-2012

Pour la structure paramétrée représentée ci-dessous, on demande :

1. De déterminer et dessiner soigneusement les diagrammes de *M*, *N* et *T*, à l'aide de la **méthode de Cross** ;

2. De calculer la rotation en B avec la méthode de Cross, de 3 façons différentes ;

3. De calculer la rotation en B avec le théorème de Pasternak ;

4. De calculer le déplacement horizontal en C ;

5. De calculer le déplacement vertical au milieu de BC.

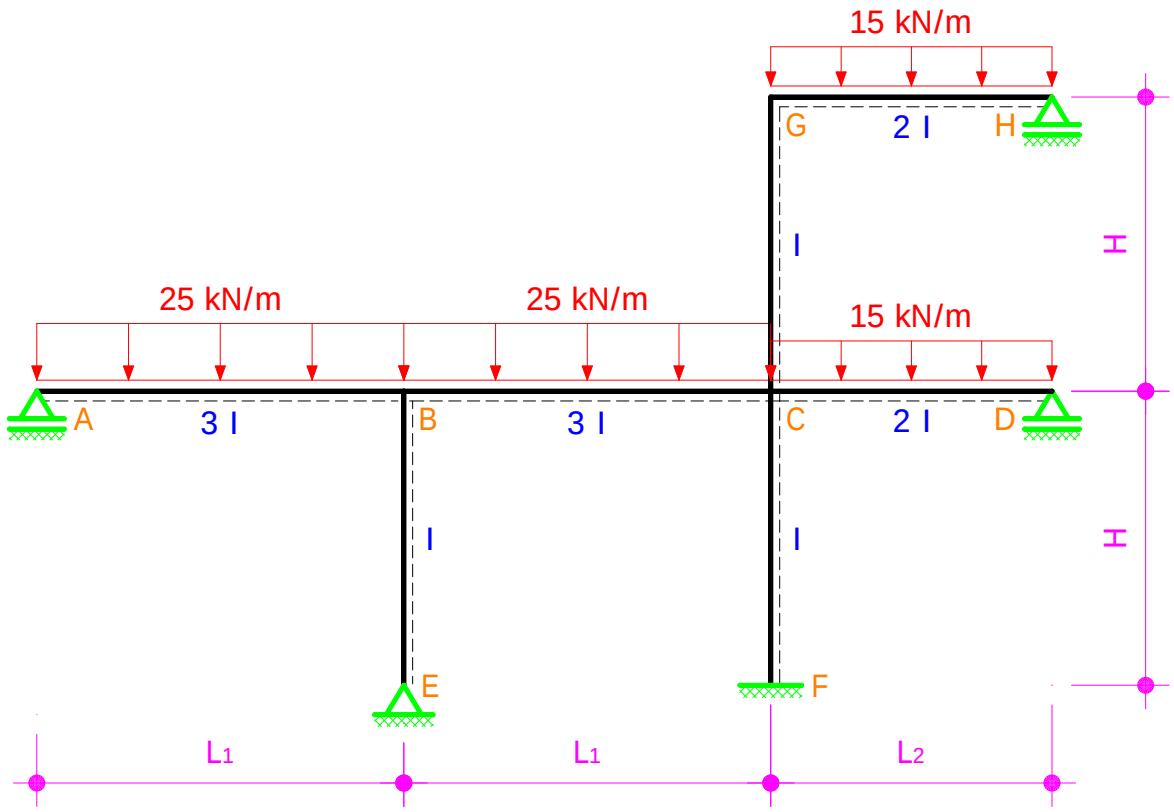
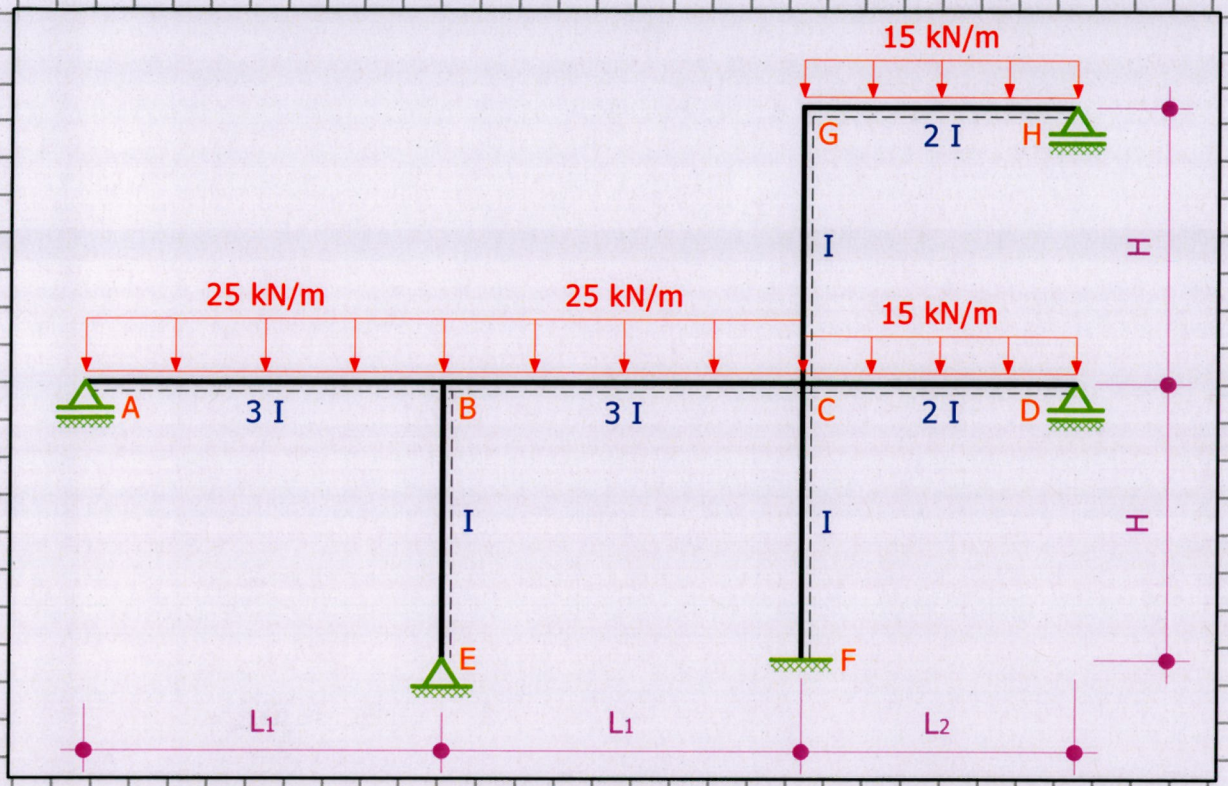


Figure 1 – Structure à étudier : géométrie et chargement

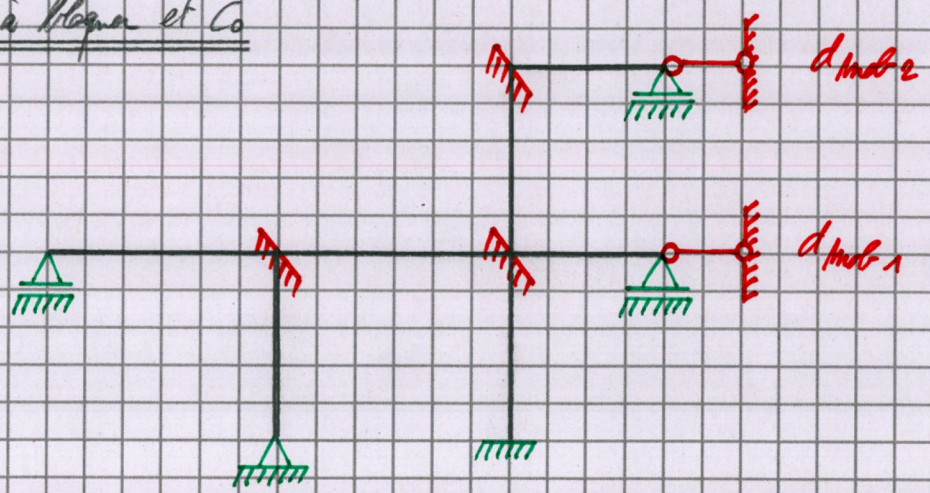
Hes·SO GENÈVE
Haute Ecole Spécialisée de Suisse occidentale



$\begin{cases} L_1 = 6\,m \\ L_2 = 4\,m \end{cases}$ $\begin{cases} H = 5\,m \\ I = 250 \cdot 10^6\,mm^4 \end{cases}$ Acier : $E = 210\,000\ N/mm^2$

Structure à nœuds déplaçables : $d_{mob} = 2$ ($\Rightarrow$ 2 inconnues hyperstatiques)

A. Choix des nœuds à bloquer et Co



B. Déterminer les coefficients de partage Π_i

$$\Sigma R_B = \left(\frac{3EI}{L}\right)_{AB} + \left(\frac{4EI}{L}\right)_{BC} + \left(\frac{3EI}{L}\right)_{BE}$$

$$= EI\left(\frac{3.3}{6} + \frac{4.3}{6} + \frac{3.1}{5}\right)$$

$$= EI\,(1,5 + 2,0 + 0,6) = \underline{EI.\,4,1}$$

$$\Pi_B^{AB} = \frac{1,5}{4,1} = \underline{0,3659} \qquad \Pi_B^{BC} = \frac{2,0}{4,1} = \underline{0,4878}$$

$$\Pi_B^{BE} = \frac{0,6}{4,1} = \underline{0,1463} \qquad \left(\sum_i \Pi_B^i = 1,0000 \;\; OK\right)$$

$$\Sigma R_C = \left(\frac{4EI}{L}\right)_{BC} + \left(\frac{3EI}{L}\right)_{CD} + \left(\frac{4EI}{L}\right)_{CG} + \left(\frac{4EI}{L}\right)_{CF}$$

$$= EI\left(\frac{4.3}{6} + \frac{3.2}{4} + \frac{4.1}{5} + \frac{4.1}{5}\right)$$

$$= EI\,(2,0 + 1,5 + 0,8 + 0,8) = \underline{EI.\,5,1}$$

$$\Pi_C^{BC} = \frac{2,0}{5,1} = \underline{0,3921} \qquad \Pi_C^{CD} = \frac{1,5}{5,1} = \underline{0,2941}$$

$$\Pi_C^{CG} = \Pi_C^{CF} = \frac{0,8}{5,1} = \underline{0,1569} \qquad \left(\sum_i \Pi_C^i = 1,0000 \;\; OK\right)$$

$$\sum R_G = \left(\frac{4EI}{L}\right)_{CG} + \left(\frac{3EI}{L}\right)_{GH}$$

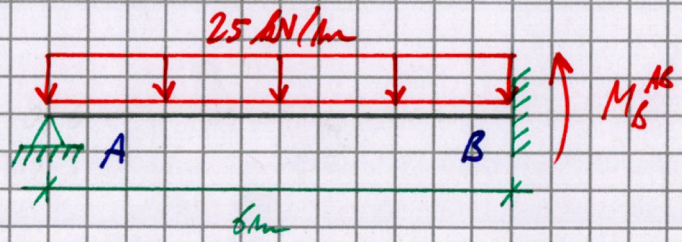
$$= EI\left(\frac{4.1}{5} + \frac{3.2}{4}\right)$$

$$= EI(0,8 + 1,5) = \underline{\underline{EI.2,3}}$$

$$\pi_G^{CG} = \frac{0,8}{2,3} = \underline{\underline{0,3478}} \qquad \pi_G^{GH} = \frac{1,5}{2,3} = \underline{\underline{0,6522}} \qquad \left(\sum_i \pi_G^i = 1,0000 \text{ OK}\right)$$

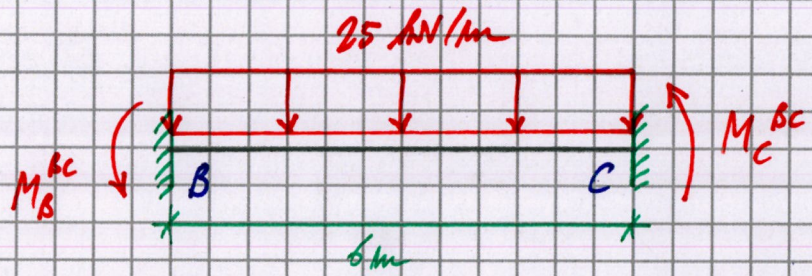
C. Déterminer les moments attaquants

* sur AB :



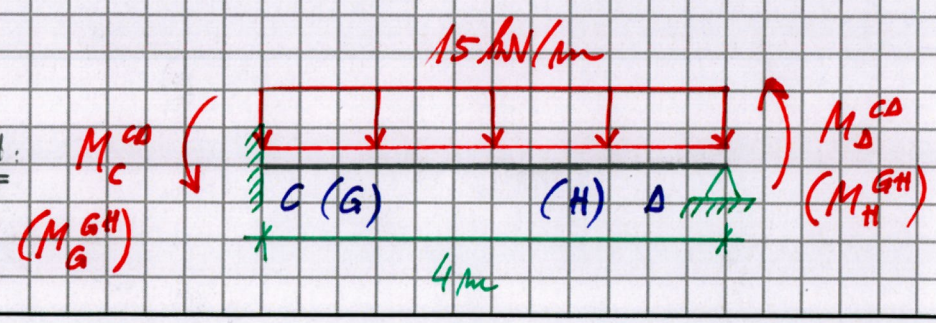
$$M_B^{AB} = -\frac{25.6^2}{8} = \underline{\underline{-112,5 \text{ kNm}}}$$

* sur BC :



$$M_B^{BC} = \frac{25.6^2}{12} = \underline{\underline{75 \text{ kNm}}} \qquad M_C^{BC} = \underline{\underline{-75 \text{ kNm}}}$$

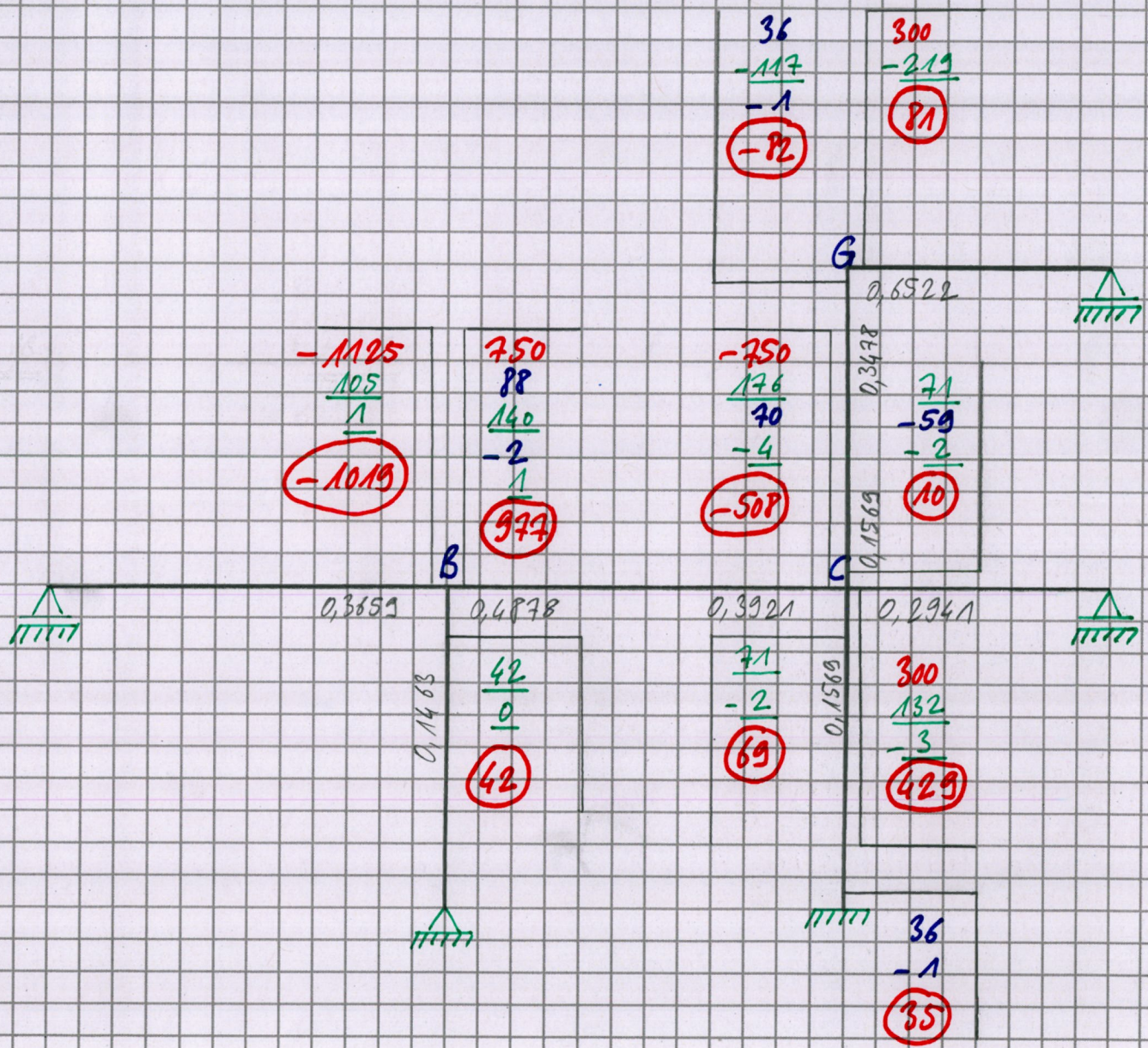
* sur CD + GH :



$M_C^{GA} = M_G^{GA} = \frac{15 \cdot 4^2}{8} = 30\ kNm$

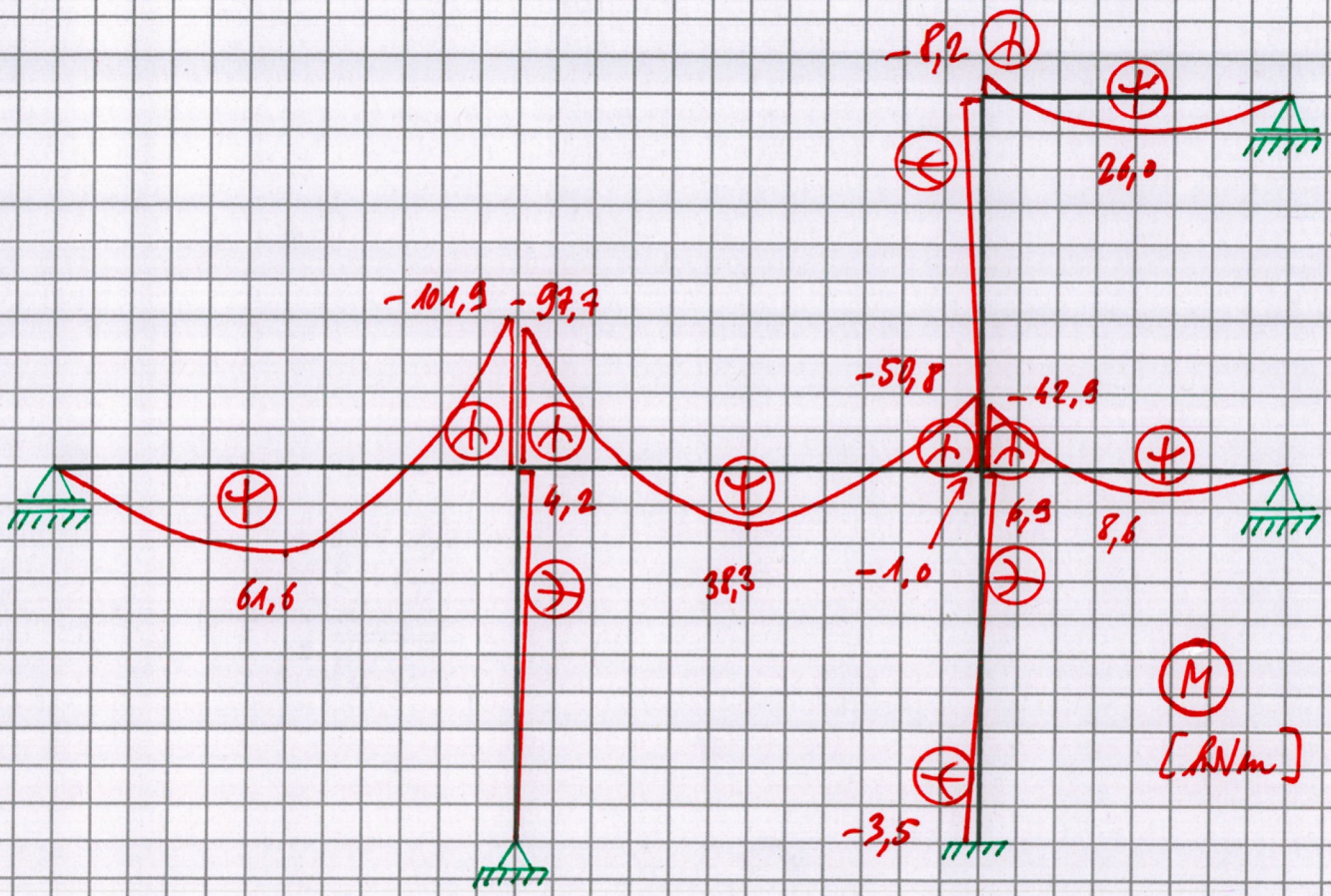
5. Appliquer le schéma de Cross

Cross x 10



Ordre de déblocage: C, B, G, C, B

E. Déterminer le diagramme des moments (convention RdM)



$$M_{\text{milieu AB}} = \frac{-101,3}{2} + \frac{25 \cdot 6^2}{8} = \underline{\underline{61,6 \text{ kNm}}}$$

$$M_{\text{milieu BC}} = \frac{-97,7 - 50,8}{2} + \frac{25 \cdot 6^2}{8} = \underline{\underline{38,3 \text{ kNm}}}$$

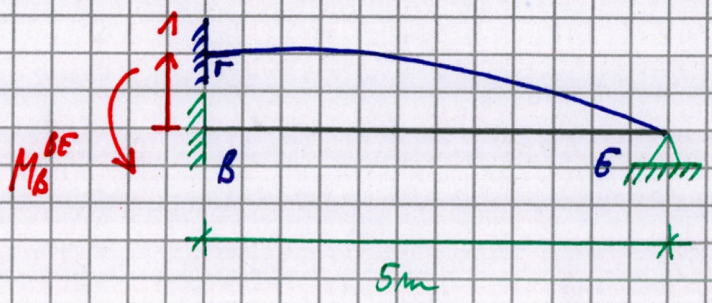
$$M_{\text{milieu CA}} = \frac{-42,9}{2} + \frac{15 \cdot 4^2}{8} = \underline{\underline{8,6 \text{ kNm}}}$$

$$M_{\text{milieu GH}} = \frac{-8,1}{2} + \frac{15 \cdot 4^2}{8} = \underline{\underline{26,0 \text{ kNm}}}$$

* Cas de la structure sous $d_{mod-1} = 1$:

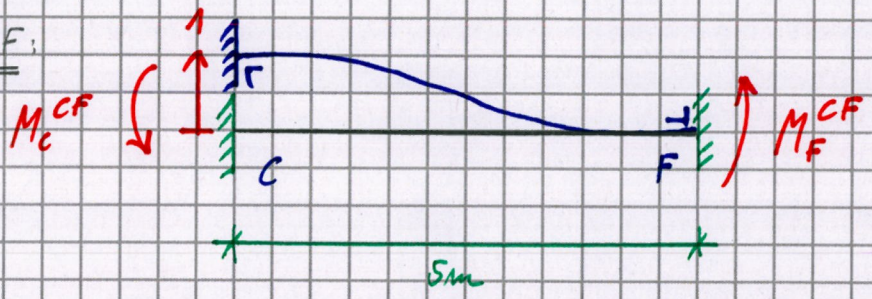
C. Déterminer les moments attaquants :

* Sur BE :



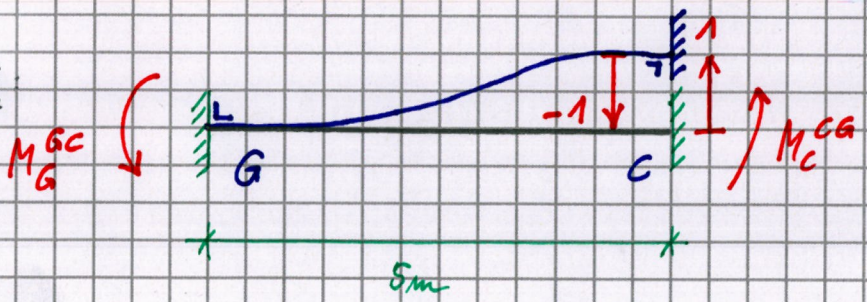
$$M_B^{BE} = \left(\frac{3EI}{L^2}\right)_{AB} \cdot 1 = EI \cdot \frac{3}{5^2} = \underline{\underline{0{,}12\ EI}}$$

* Sur CF :



$$M_C^{CF} = M_F^{CF} = \left(\frac{6EI}{L^2}\right)_{CF} \cdot 1 = EI \cdot \frac{6}{5^2} = \underline{\underline{0{,}24\ EI}}$$

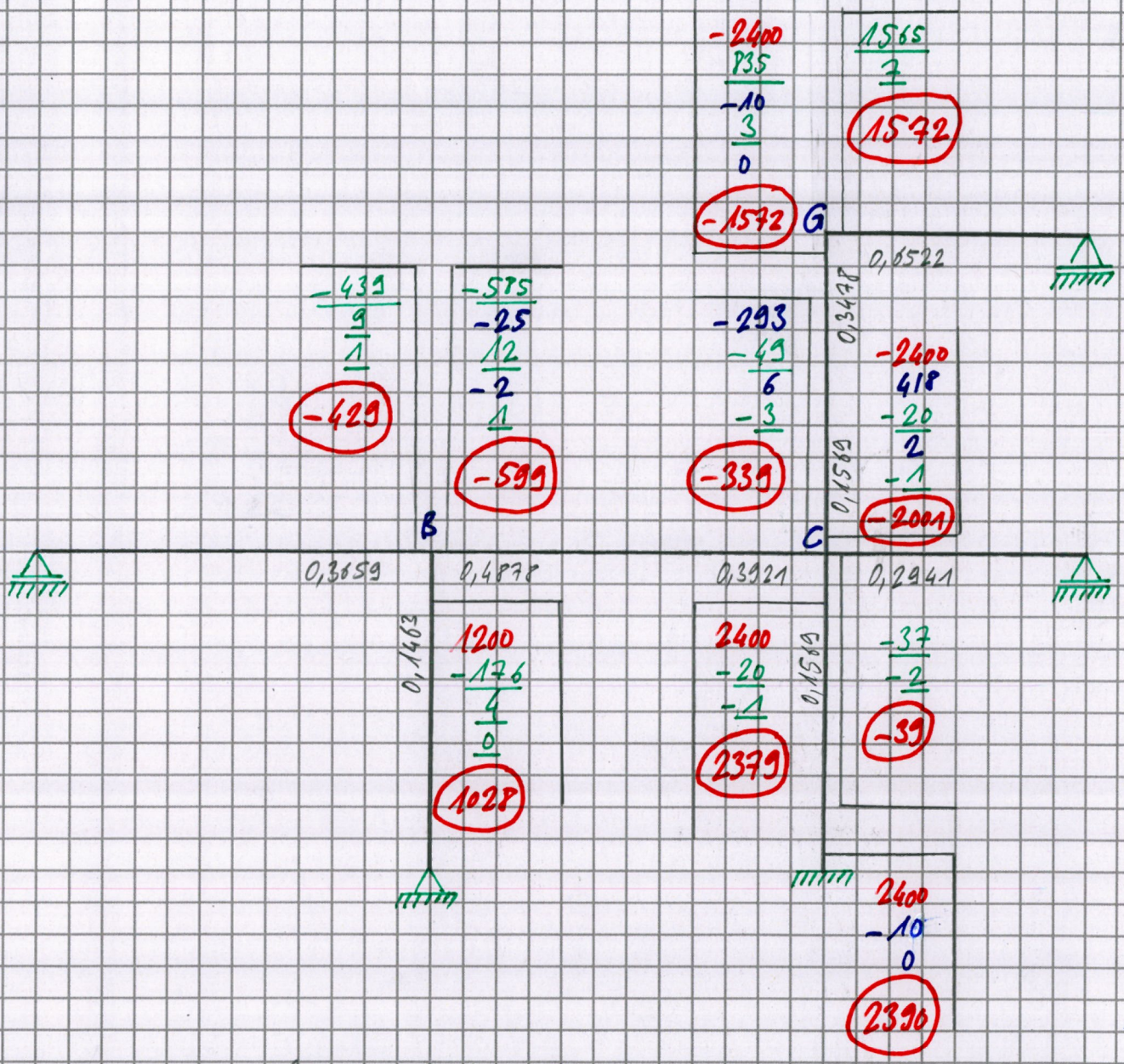
* Sur CG :



$$M_G^{GC} = M_C^{CG} = \left(\frac{6EI}{L^2}\right)_{GC} \cdot -1 = \underline{\underline{-0{,}24\ EI}}$$

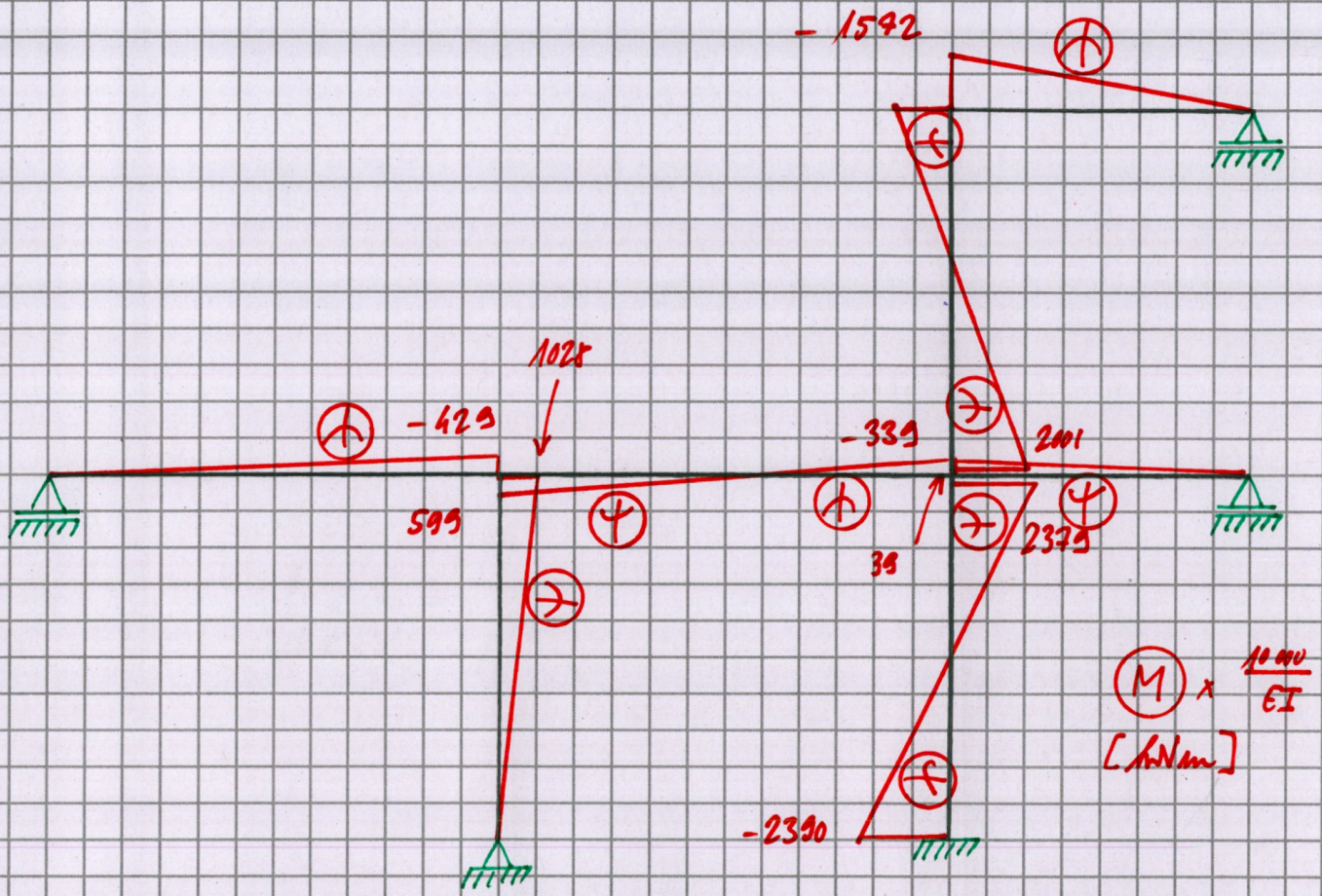
5. Appliquer le schéma de Cross

Cross x 10000. $\frac{1}{EI}$



ordre de déblocage: G, B, C, B, G, C, B

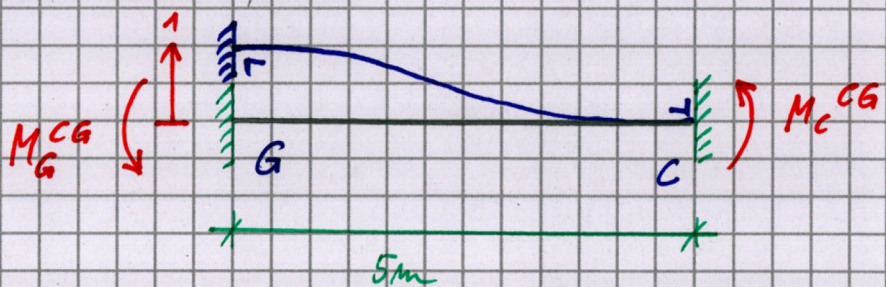
E. Déterminer le diagramme des moments (convent° RdM)



* Cas de la structure sous $d_{mob\,2} = 1$:

C. Déterminer les moments attaquants

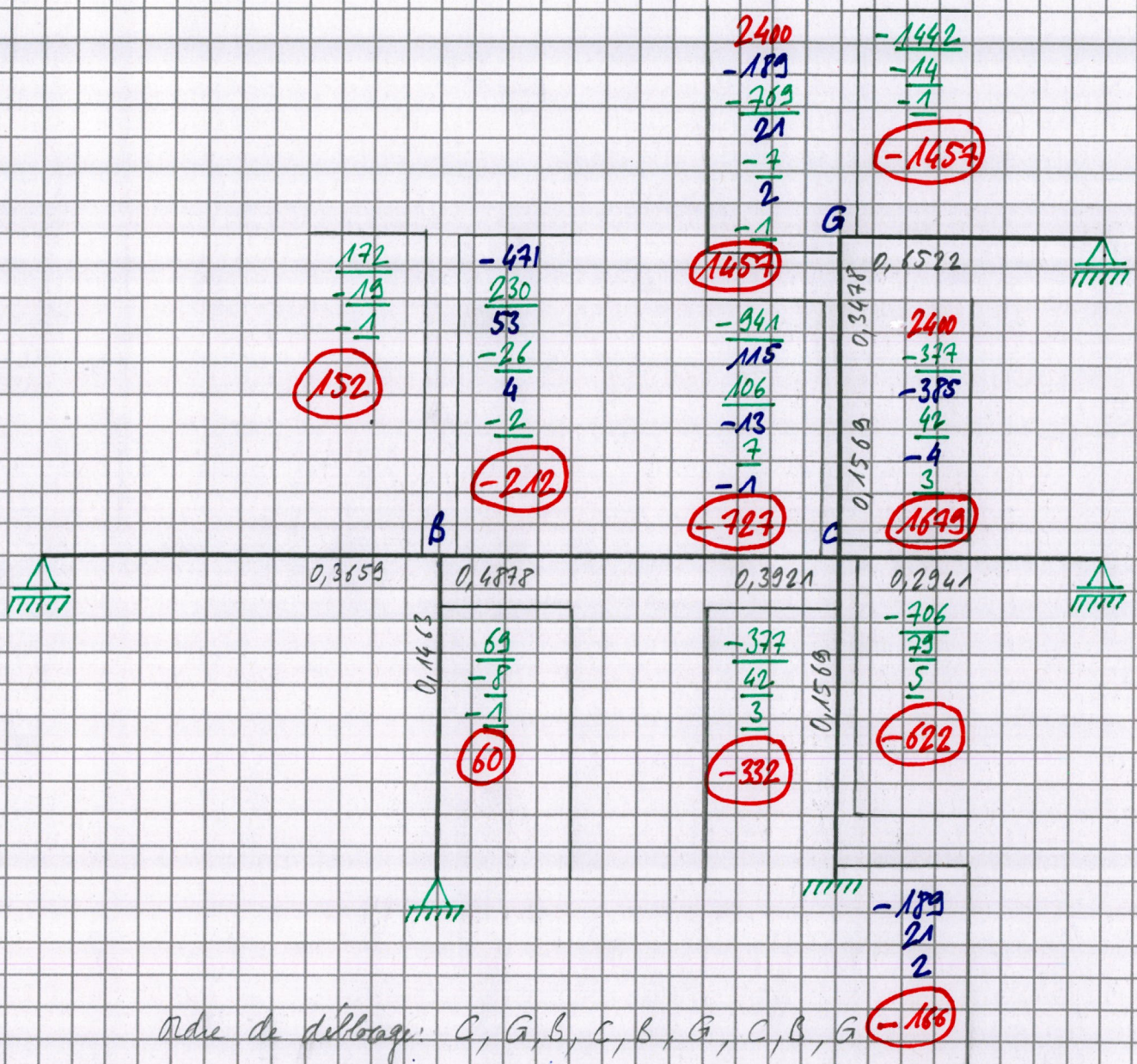
* Sur CG :



$$M_G^{CG} = M_C^{CG} = \left(\frac{6\,EI}{L^2}\right)_{GC} \cdot 1 = \underline{\underline{0{,}24 \cdot EI}}$$

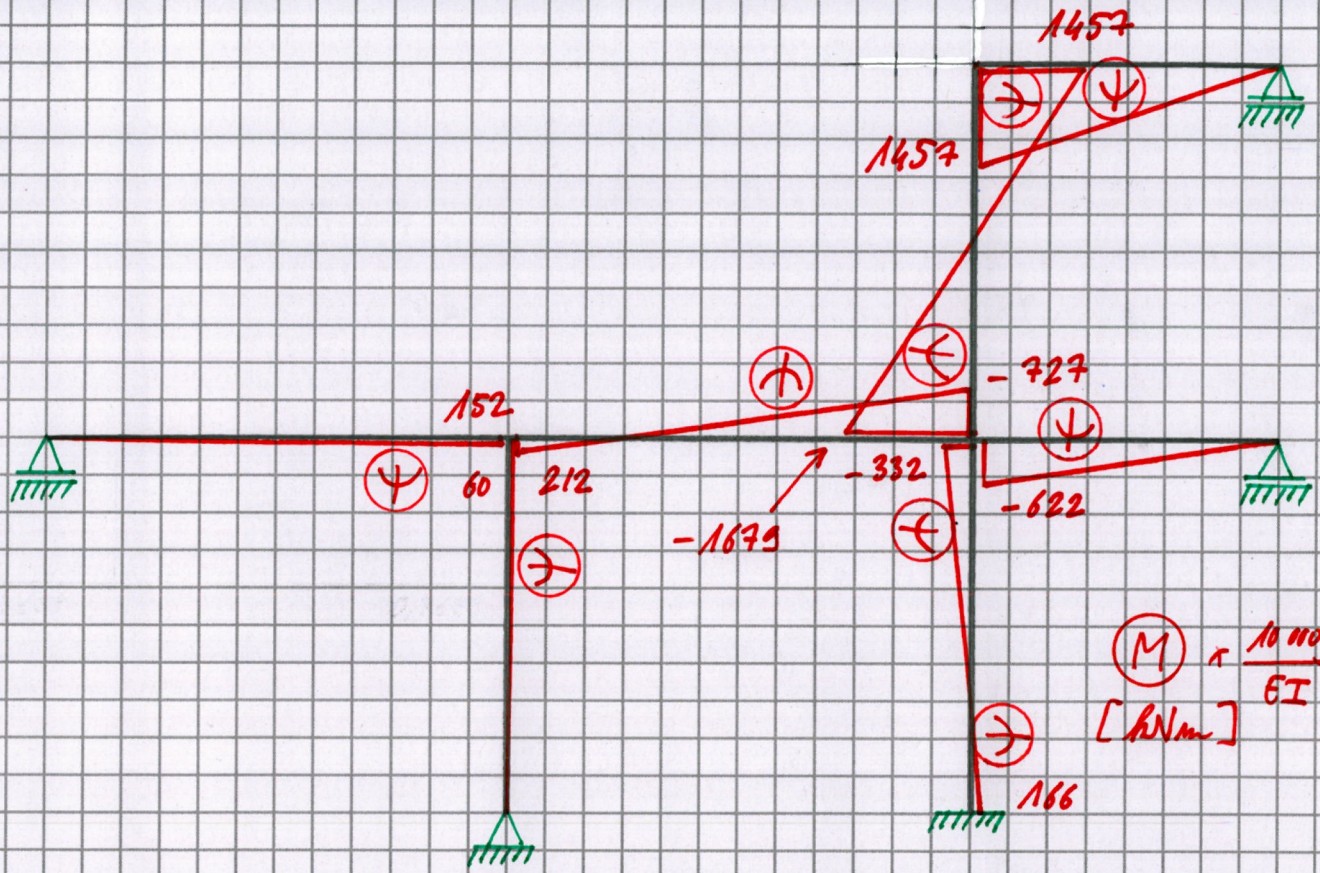
5. Appliquer le schéma de Cross

Cross × 10 000. $\frac{1}{EI}$



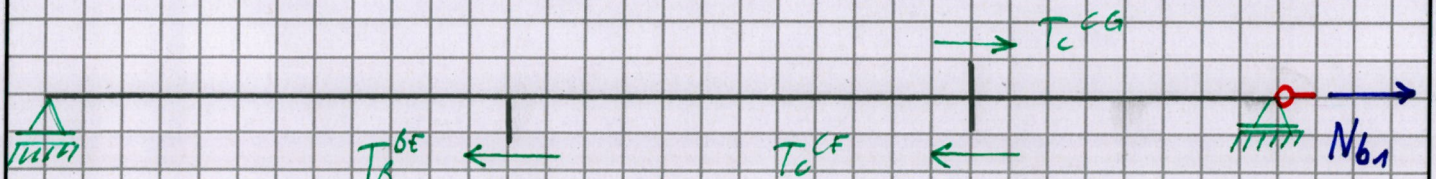
Ordre de déblocage: C, G, B, C, B, G, C, B, G

E. Déterminer le diagramme des moments (convent° RdM)



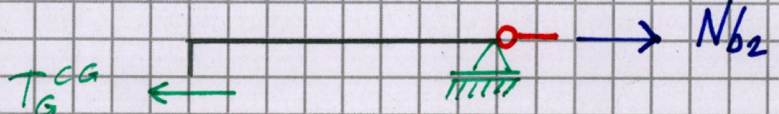
* Superposition des effets : calcul des efforts normaux dans les bielles

- d mdb1 :



$$N_{b1} = T_B^{BE} + T_C^{CF} - T_C^{CG}$$

- d mdb2 :



$$N_{b2} = T_G^{CG}$$

- Cas du système Gsm 0 :

$$T_B^{BE} = \frac{4,2}{5} = \underline{0,84 \text{ kN}} \qquad T_C^{CF} = \frac{6,9+3,5}{5} = \underline{2,08 \text{ kN}}$$

$$T_C^{CG} = \frac{-8,2+1,0}{5} = \underline{-1,44 \text{ kN}}$$

$$N_{b1,0} = 0,84 + 2,08 + 1,44 = \underline{4,36 \text{ kN}}$$

- Cas du système Gsm 1 :

$$T_B^{CG} = \frac{EI}{10\,000}\left(\frac{1028}{5}\right) = \underline{20,56\,10^{-3}.\,EI}$$

$$T_C^{CF} = \frac{EI}{10\,000}\left(\frac{2379+2390}{5}\right) = 95,38.10^{-3}.EI$$

$$T_C^{CG} = \frac{EI}{10\,000}\left(\frac{-1572-2001}{5}\right) = -71,46.10^{-3}.EI$$

$$Nb_{1,1} = (20,56 + 95,38 + 71,46).10^{-3}.EI = 187,4.10^{-3}.EI$$

- Cas du système Cas 2:

$$T_B^{BE} = \frac{EI}{10\,000}\left(\frac{60}{5}\right) = 1,2.10^{-3}.EI$$

$$T_C^{CF} = \frac{EI}{10\,000}\left(\frac{-322-166}{5}\right) = -9,76.10^{-3}.EI$$

$$T_C^{CG} = \frac{EI}{10\,000}\left(\frac{1457+1679}{5}\right) = 62,72.10^{-3}\,EI$$

$$Nb_{1,2} = (1,2 - 9,76 - 62,72).10^{-3}.EI = -71,28.10^{-3}.EI$$

$$Nb_{2,0} = -1,44\ kN$$

$$Nb_{2,1} = -71,46.10^{-3}.EI$$

$$Nb_{2,2} = 62,72.10^{-3}.EI$$

$\Rightarrow$ Système à résoudre:

$$\begin{cases} 4,36.10^{3} + X_1.187,4.10^{-3}.EI - X_2.71,28.10^{-3}.EI = 0 \\ -1,44.10^{3} - X_1.71,46.10^{-3}.EI + X_2.62,72.10^{-3}.EI = 0 \end{cases}$$

$$\begin{cases} X_1 = -4,885\cdot10^{-4}\,m = -0,4885\,mm \;(\leftarrow) \\ X_2 = -1,193\cdot10^{-4}\,m = -0,1193\,mm \;(\leftarrow) \end{cases}$$

- $M_B^{BE} = 4,2\cdot10^3 + \left(-4,885\cdot10^{-4}\cdot 1028 - 1,193\cdot10^{-4}\cdot 60\right)\cdot\frac{EI}{10\,000} = 1,53\ kNm$
- $M_F^{CF} = -3,5\cdot10^3 + \left(4,885\cdot10^{-4}\cdot 2390 - 1,193\cdot10^{-4}\cdot 166\right)\cdot\frac{EI}{10\,000} = 2,53\ kNm$
- $M_C^{CF} = 6,9\cdot10^3 + \left(-4,885\cdot10^{-4}\cdot 2379 + 1,193\cdot10^{-4}\cdot 332\right)\cdot\frac{EI}{10\,000} = 1,01\ kNm$
- $M_C^{CG} = -1,0\cdot10^3 + \left(-4,885\cdot10^{-4}\cdot 2001 + 1,193\cdot10^{-4}\cdot +1679\right)\cdot\frac{EI}{10\,000} = -5,1\ kNm$
- $M_B^{AB} = -101,9\cdot10^3 + \left(4,885\cdot10^{-4}\cdot 429 - 1,193\cdot10^{-4}\cdot 152\right)\cdot\frac{EI}{10\,000} = -100,9\ kNm$
- $M_B^{BC} = -97,7\cdot10^3 + \left(-4,885\cdot10^{-4}\cdot 599 - 1,193\cdot10^{-4}\cdot 212\right)\cdot\frac{EI}{10\,000} = -99,4\ kNm$
- $M_C^{BC} = -50,8\cdot10^3 + \left(4,885\cdot10^{-4}\cdot 339 + 1,193\cdot10^{-4}\cdot 727\right)\cdot\frac{EI}{10\,000} = -49,5\ kNm$
- $M_C^{CD} = -42,9\cdot10^3 + \left(-4,885\cdot10^{-4}\cdot 38 + 1,193\cdot10^{-4}\cdot 622\right)\cdot\frac{EI}{10\,000} = -43,3\ kNm$
- $M_G = -8,2\cdot10^3 + \left(4,885\cdot10^{-4}\cdot 1572 - 1,193\cdot10^{-4}\cdot 1457\right)\cdot\frac{EI}{10\,000} = -5,1\ kNm$

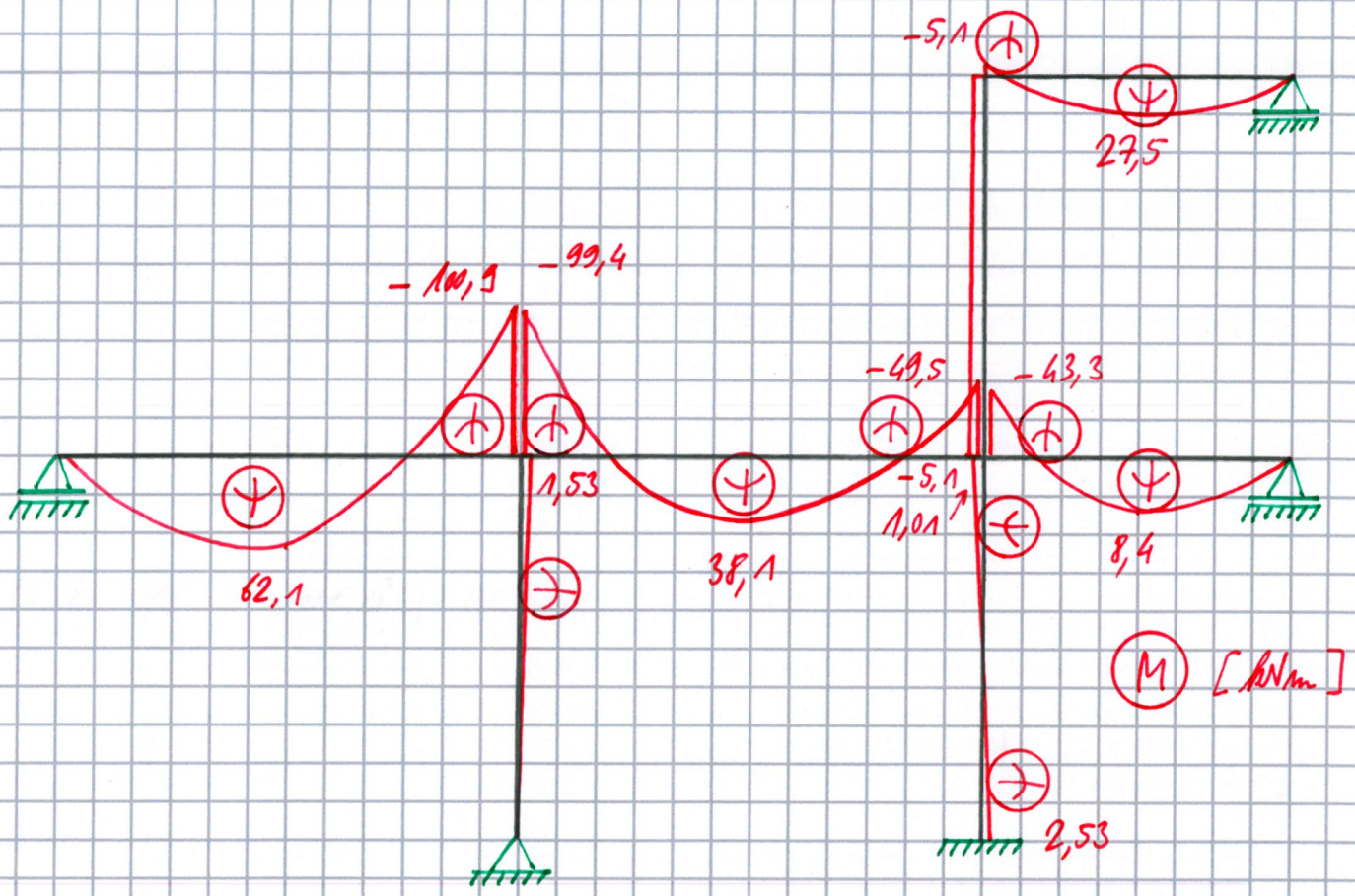
$$M_{milieu\,AB} = \frac{-100,9}{2} + \frac{25\cdot 8^2}{8} = 62,1\ kNm$$

$$M_{milieu\,BC} = \frac{-99,4-49,5}{2} + \frac{25\cdot 8^2}{8} = 38,1\ kNm$$

$$M_{milieu\,CD} = \frac{-43,3}{2} + \frac{15\cdot 4^2}{8} = 8,4\ kNm$$

$$M_{milieu\,GH} = \frac{-5,1}{2} + \frac{15\cdot 4^2}{8} = 27,5\ kNm$$

* Diagramme des moments finaux (convention RdM)



* Diagramme d'effort tranchant :

- sur AB :

$$T_A^{AB} = \frac{-100,9}{6} + \frac{25 \cdot 6}{2} = 58,2 \text{ kN}$$

$$T_B^{AB} = \frac{-100,9}{6} - \frac{25 \cdot 6}{2} = -91,8 \text{ kN}$$

- sur BC :

$$T_B^{BC} = \frac{-49,5 + 99,4}{6} + \frac{25 \cdot 6}{2} = 83,3 \text{ kN}$$

$$T_C^{BC} = \frac{-49,5 + 99,4}{6} - \frac{25 \cdot 6}{2} = -66,7 \text{ kN}$$

- Barre CD : $T_C^{CD} = \frac{43,3}{4} + \frac{15.4}{2} = 40,8 \text{ kN}$

  $T_D^{CD} = \frac{43,3}{4} - \frac{15.4}{2} = -19,2 \text{ kN}$

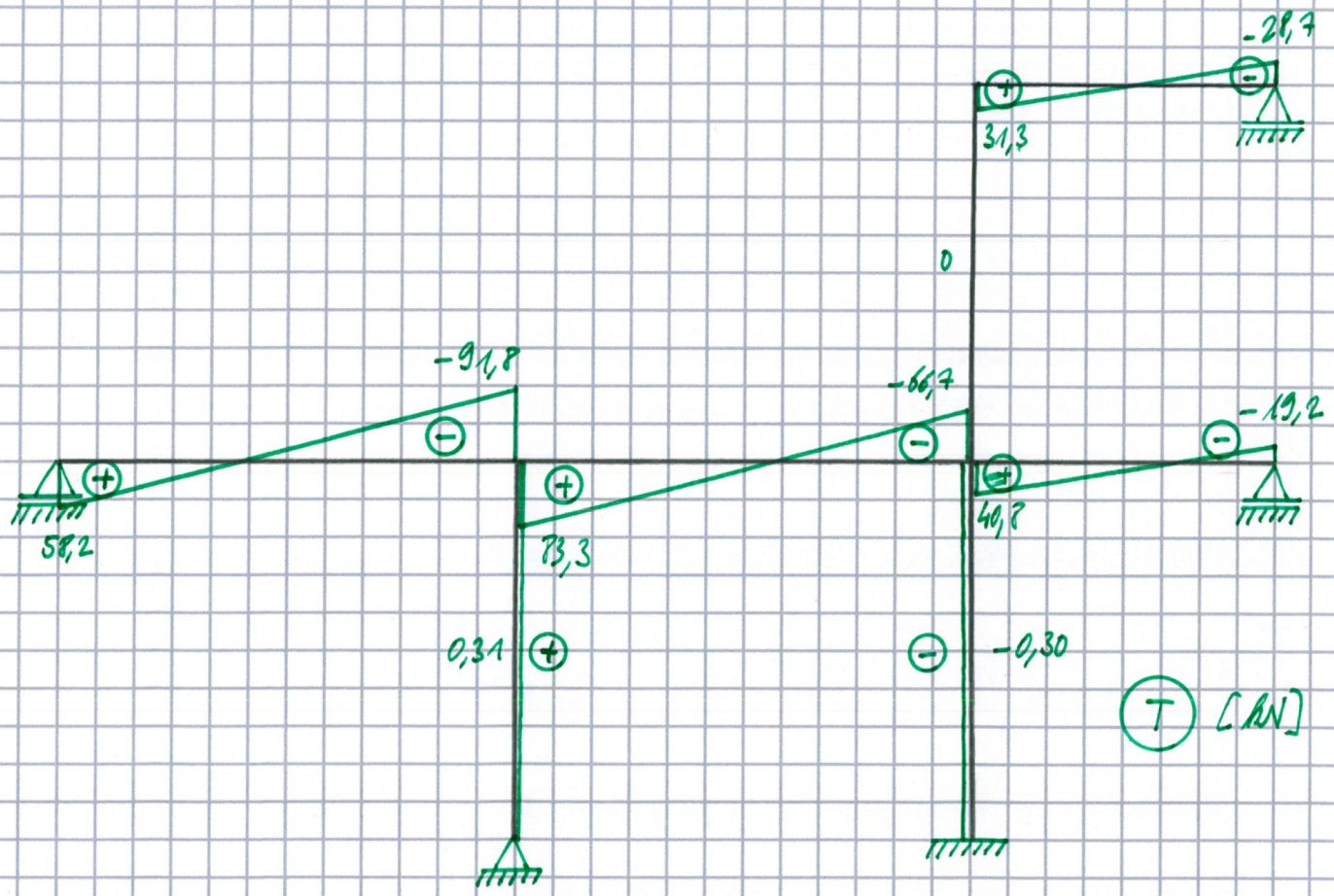
- Barre GH : $T_G^{GH} = \frac{5,1}{4} + \frac{15.4}{2} = 31,3 \text{ kN}$

  $T_H^{GH} = \frac{5,1}{4} - \frac{15.4}{2} = -28,7 \text{ kN}$

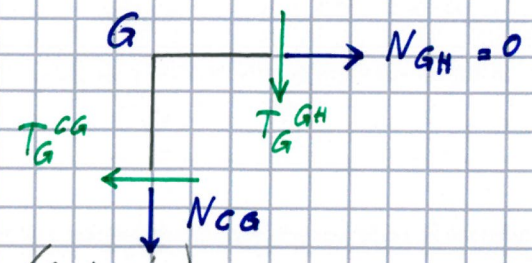
- Barre BE : $T_B^{BE} = T_E^{BE} = \frac{1,53}{5} = 0,31 \text{ kN}$

- Barre CF : $T_C^{CF} = T_F^{CF} = \frac{1,01 - 2,53}{5} = -0,30 \text{ kN}$

- Barre CG : $T_C^{CG} = T_G^{CG} = 0 \text{ kN}$

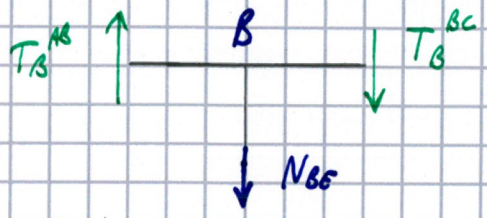


* Diagramme d'effort normal :



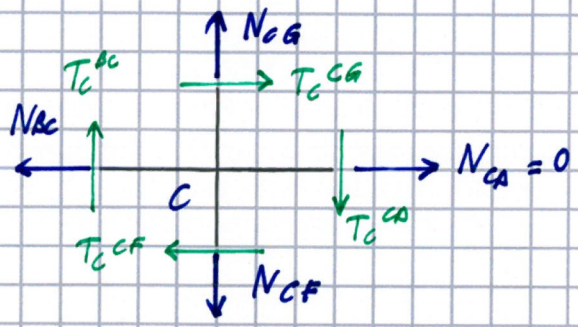
- au nœud G :

$$N_{CG} = -T_G^{GH} = \underline{-31,3\ kN} \text{ (compression)}$$



- au nœud B :

$$N_{BG} = T_B^{AB} - T_B^{BC} = -91,8 - 83,3$$
$$= \underline{-175,1\ kN} \text{ (compression)}$$

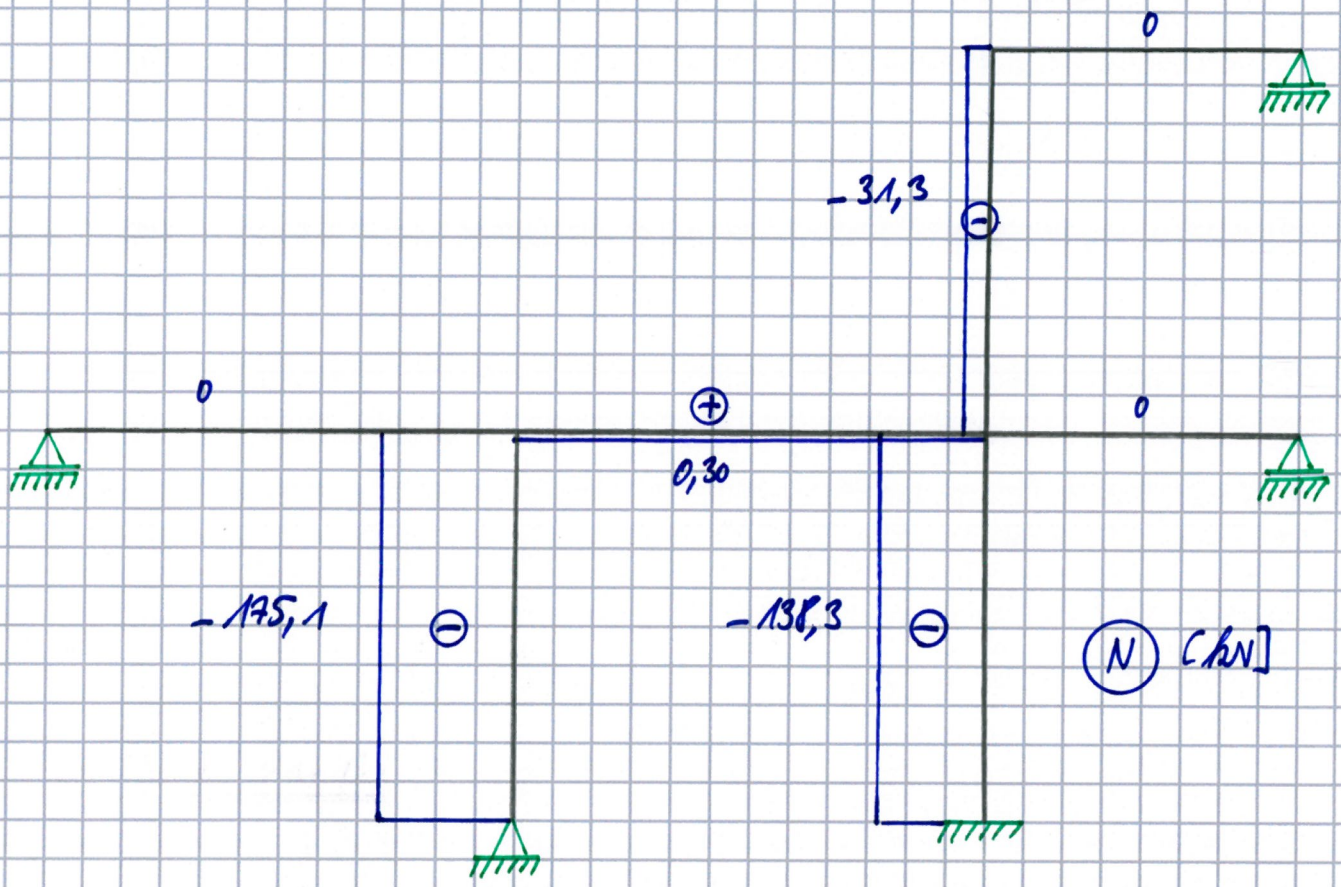


- au nœud C :

$$N_{CF} = N_{CG} + T_C^{BC} - T_C^{CA}$$
$$= -31,3 - 66,7 - 40,8$$
$$= \underline{-138,3\ kN} \text{ (compression)}$$

$$N_{BC} = T_C^{CG} - T_C^{CF} = 0 + 0,30 = \underline{0,30\ kN} \text{ (traction)}$$

(vérification au nœud B : $N_{BC} = T_B^{BC} = 0,31\ kN$ OK)



2. Calculer la rotation en B avec la méthode de Cross (3 façons différentes)

* Avec AB:

$$\phi_B^{AB}{}_{CM0} = \frac{(105+1).\ 0,1.10^3}{3.\ \frac{2,1.10^4.\ 3.250.10^{-6}}{6}} = \underline{1,346.10^{-4}\ rad.}$$

$$\phi_B^{AB}{}_{CM1} = \frac{(-439+9+1).\ \frac{EI}{10\,000}}{3.\ \frac{EI.3}{6}} = \underline{-28,6.10^{-3}\ rad.}$$

$$\phi_B^{AB}{}_{CM2} = \frac{(172-18-1).\ \frac{EI}{10\,000}}{3.\ \frac{EI.3}{6}} = \underline{10,1.10^{-3}\ rad.}$$

$$\phi_B^{1B} = 1,346.10^{-4} + 4,885.10^{-4}.28,6.10^{-3} - 1,193.10^{-4}.10,1.10^{-3}$$
$$= 1,474.10^{-4}\ rad = \underline{0,1474\ mrad}.$$

* Avec BC :

$$\phi_{B0}^{BC} = \frac{(140+1).0,1.10^{3}}{4.\frac{2,1.10^{11}.3.250.10^{-6}}{6}} = \underline{1,343.10^{-4}\ rad} \quad OK$$

$$\phi_{B\ com1}^{BC} = \frac{(-585+12+1).\frac{EI}{10\,000}}{4.\frac{EI.3}{6}} = \underline{-28,6.10^{-3}\ rad} \quad OK$$

$$\phi_{B\ com2}^{BC} = \frac{(230-26-2).\frac{EI}{10\,000}}{4.\frac{EI.3}{6}} = \underline{10,1.10^{-3}\ rad} \quad OK$$

$$\phi_B^{BC} = 1,343.10^{-4} + 4,885.10^{-4}.28,6.10^{-3} - 1,193.10^{-4}.10,1.10^{-3}$$
$$= 1,471.10^{-4}\ rad = \underline{0,1471\ mrad} \quad OK \quad (0,2\,\%)$$

* Avec BE :

$$\phi_{B\ com0}^{BE} = \frac{42.0,1.10^{3}}{3.\frac{2,1.10^{11}.250.10^{-6}}{5}} = \underline{1,333.10^{-4}\ rad} \quad OK$$

$$\phi_{B\ com1}^{BE} = \frac{(-176+4).\frac{EI}{10\,000}}{3.\frac{EI}{5}} = \underline{-28,6.10^{-3}\ rad} \quad OK$$

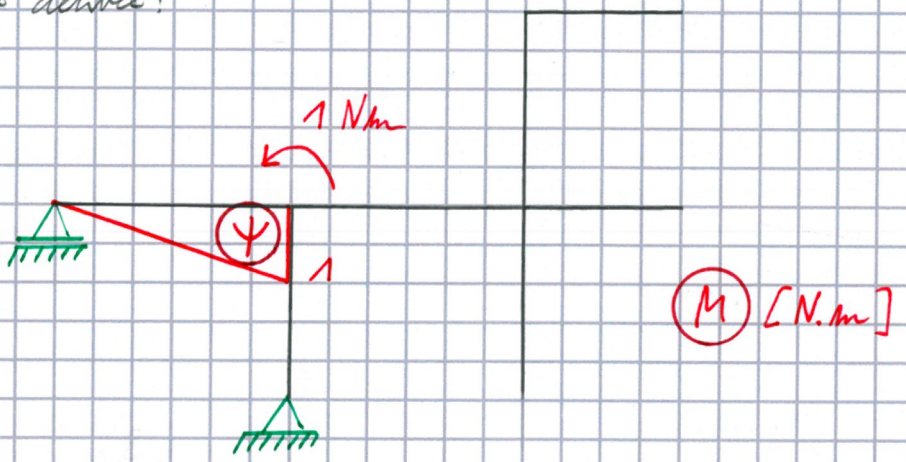
$$\phi_{B\ com2}^{BE} = \frac{(69-8-1).\frac{EI}{10\,000}}{3.\frac{EI}{5}} = \underline{10,0.10^{-3}\ rad} \quad OK$$

$$\phi_B^{BE} = 1,333.10^{-4} + 4,885.10^{-4}.28,6.10^{-3} - 1,193.10^{-4}.10,0.10^{-3}$$

$$= 1,461.10^{-4}\ \text{rad} = \underline{\underline{0,1461\ \text{mrad}}} \quad \text{OK}$$

3. Calcul de la rotation en B avec le théorème de Pasternak

=> structure dérivée :



$$\phi_B = \frac{6}{2,1.10^{11}.250.10^{-6}.3} \cdot \left(\underset{1}{\triangle} \cdot \left(\overset{-100,9.10^3}{\triangle} + \underset{112,25.10^3}{\triangle} \right) \right)$$

$$= 3,810.10^{-8}.\frac{1}{3}.1.10^3.(-100,9 + 112,25) = 1,441.10^{-4}\ \text{rad}$$

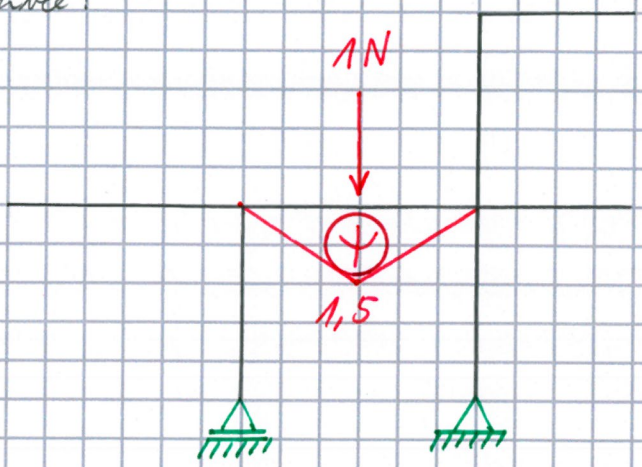
$$= \underline{\underline{0,1441\ \text{mrad}}} \ (2\%, \text{OK})$$

4. Calcul du déplacement horizontal en C

$$d_{HC} = X_1 = -4,885.10^{-4}\ \text{m} = \underline{\underline{-0,4885\ \text{mm}}} \ (\text{vers la gauche})$$

5. Calcul du déplacement vertical au milieu de BC :

⇒ Méthode dérivée :



$$f_{milieu\ BC} = \frac{6}{2,1.10^{11}.3.250.10^{-6}} \cdot \frac{1}{1,5} \cdot \left(\underbrace{-99,4.10^{-3}}_{+} + \underbrace{-49,5.10^{-3}}_{+} + \underbrace{112,25.10^{-3}}_{} \right)$$

$$= 3,810.10^{-8} \cdot 1,5.10^{3} \cdot \left(\frac{1}{4}.-99,4 - \frac{1}{4}.49,5 + \frac{5}{12}.112,25 \right)$$

$$= 5,455.10^{-4}\ m = \underline{0,5455\ mm} \quad (\text{vers le bas})$$